Quiver Chern-Simons theories and 3-algebra orbifolds

Nakwoo Kim

Department of Physics and Research Institute of Basic Science,

Kyung Hee University, Seoul 130-701, Korea[*]

(Dated: December 30, 2009)

Abstract

We attempt to derive quiver Chern-Simons-matter theories from the Bagger-Lambert theory with Nambu bracket, through an orbifold prescription which effectively induces a dimensional reduction of the internal space for 3-algebra. We consider M2-branes on an $\mathcal{N} = 4$ orbifold $\mathbb{C}^2/\mathbb{Z}_k \times \mathbb{C}^2$, and compare the result with the so-called dual Aharony-Bergman-Jafferis-Maldacena model, proposed recently by Hanany, Vegh, and Zaffaroni. Unlike the $\mathcal{N} = 6$ example $\mathbb{C}^4/\mathbb{Z}_k$, we find ambiguities in the matrix regularization.

PACS numbers: 11.25.-w

arXiv:0912.5418v1 [hep-th] 30 Dec 2009

[*]Electronic address: nkim@khu.ac.kr

I. INTRODUCTION

The AdS/CFT correspondence [1] suggests that there exist a three dimensional superconformal field theory which is dual to M-theory in $AdS_4 \times S^7$ background. For a long time the gauge field theory for multiple membranes has remained a mystery, but recently a lot of progress has been made through the study of Chern-Simons-matter theory as a candidate description.

Bagger and Lambert [2, 3, 4], and also independently Gustavsson [5], constructed new maximally supersymmetric gauge theories in three dimensions. These theories are based on a 3-algebra: unlike the familiar Yang-Mills symmetry, 3-algebra naturally induces quartic Yukawa-type interaction, while scalar fields exhibit sixth-order potential. The relevance of 3-algebra in membrane theory is justified by the Myers effect [6], which predicts multiple membranes in external field polarize into M5-branes.

When one combines 3-algebra with maximal supersymmetry, the mathematical consistency condition turned out to be too stringent. There exist only one positive-definite, finite dimensional 3-algebra [7, 8]. And then the entire Bagger-Lambert-Gustavsson (BLG) theory simplifies to Chern-Simons-matter theory with $SU(2) \times SU(2)$ gauge symmetry, with levels k and $-k$, and four bifundamental chiral multiplets with quartic superpotential.

On the other hand, Aharony, Bergman, Jafferis and Maldacena (ABJM) constructed new $\mathcal{N} = 6$ superconformal Chern-Simons-matter theories, which describe multiple membranes on orbifold $\mathbb{C}^4/\mathbb{Z}_k$ [9]. Instead of relying on 3-algebra as a new dynamical principle, they started with certain IIB brane configurations and utilized string duality relations and identified the gauge field theory living on brane intersections. Unlike BLG theory, the ABJM model can describe an arbitrary number of membranes: when there are N M2-branes, the theory has $U(N) \times U(N)$ gauge symmetry with level k and $-k$.

It is then natural to ask how to generalize to different orbifolds. One can try to devise sophisticated brane configurations which would lead to less supersymmetric orbifolds [10]. Or one scans the set of quiver Chern-Simons-matter theories to discover new examples of duality between gauge theory and geometric singularities [11]. More systematically one should make use of brane tiling [12] or brane crystal [13, 14] techniques.

In this article we take a different route. We start with the infinite dimensional 3-algebra, which is the Nambu bracket of functions on T^3. The physical meaning of such theory with

an infinite number of fields is rather unclear, although one might conceive it is suggestive of higher-dimensional M-theory branes [15, 16, 17, 18]. In this paper we use the Nambu bracket theory as a technical tool and impose a 3-algebra version of orbifold truncation which effectively reduces $T^3 \to T^2$. This technique has been applied to $\mathbb{C}^4/\mathbb{Z}_k$ and correctly generated the large-N limit of the ABJM model [19].

We study specifically a $\mathcal{N}=4$ orbifold $\mathbb{C}^2/\mathbb{Z}_k \times \mathbb{C}^2$ in this article. In fact a quiver gauge theory has been already proposed for this example, and is dubbed a dual-ABJM model [11]. It again possesses $U(N)\times U(N)$ with levels $k, -k$, but with a different matter content. One hypermultiplet is in bi-fundamental, and the other is in adjoint representation of $U(N)$.

Although the mesonic vacuum moduli space is certainly $\mathbb{C}^2/\mathbb{Z}_k \times \mathbb{C}^2$, the correspondence is rather subtle. First of all, the field theory itself has only $\mathcal{N}=3$ supersymmetry, and the scalar potential fails to exhibit $SO(4)$ symmetry [20]. The superconformal index computation largely agrees with supergravity result but not completely, although it is still open that the twisted sector contribution might be responsible for the mismatch [21].

Using the Nambu bracket we will see that one can reproduce the dual-ABJM model, but there is a caveat. For the adjoint matter one has to ignore terms which converge more slowly than in the rest of the action, when taking the large-N limit. Our analysis hopefully provides a different perspective for the validity of dual-ABJM conjecture.

In Sec.II we review the BLG theory and the Nambu bracket orbifold procedure. Sec.III is the main part which studies $\mathbb{C}^2/\mathbb{Z}_k \times \mathbb{C}^2$ orbifolds. We conclude in Sec.IV with discussions.

II. FROM 3-ALGEBRA TO QUIVER GAUGE THEORY

We start by briefly reviewing the Bagger-Lambert theory [2, 3, 4] and the orbifold proposal made in Ref. [19]. The $D=3$, $\mathcal{N}=8$ superconformal field theory [4] is based on a 3-algebra. For basis vectors T^a, the structure constants f^{abcd} are defined through a 3-product as

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d. \tag{1}$$

We are particularly interested in the Nambu bracket in this paper. The elements of 3-algebra are scalar functions on a compact Riemannian 3-manifold Σ. With metric g and coordinates σ_i, $i=1,2,3$, the Nambu bracket is

$$[X, Y, Z] = \frac{1}{\sqrt{g}}\epsilon^{ijk}\partial_i X \partial_j Y \partial_k Z. \tag{2}$$

In this paper we choose $\Sigma = T^3$ with $g_{ij} = \delta_{ij}$, and set the period of σ_i to be 2π. The inner product of two elements in this 3-algebra is then naturally defined as

$$(X, Y) \equiv \mathrm{tr}(X^* Y) = \frac{1}{(2\pi)^3} \int d^3\sigma X^* Y. \tag{3}$$

If we choose the basis vectors $X_{\vec{M}}$ to be associated with the Fourier mode $e^{i\vec{M}\cdot\vec{\sigma}}$, the structure constants are easily computed

$$f_{\vec{N}_1,\vec{N}_2,\vec{N}_3,\vec{N}_4} = i\kappa(\vec{N}_1 \times \vec{N}_2 \cdot \vec{N}_3)\,\delta(\vec{N}_1 + \vec{N}_2 + \vec{N}_3 + \vec{N}_4), \tag{4}$$

where κ is a tunable coupling constant.

As usual with supersymmetric gauge theory, the Bagger-Lambert action is determined once we specify the matter content, the gauge transformation rules and the superpotential. There are four chiral multiplets, and the four complex scalar fields $Z^I, I = 1, 2, 3, 4$ couple to the gauge fields as follows

$$D_\mu Z^I_a = \partial_\mu Z^I_a - \tilde{A}_\mu{}^b{}_a Z^I_b, \quad \tilde{A}_\mu{}^b{}_a = f^{cdb}{}_a A_{\mu cd}. \tag{5}$$

And the superpotential is given as

$$W = 2\,\mathrm{tr}([Z^1, Z^2, Z^3]Z^4). \tag{6}$$

Instead of working with the entire Lagrangian, we will just consider how the truncation and regularization process affect the bosonic sector, i.e. scalar fields and gauge field. We will also analyze their interactions through the gauge coupling and superpotential. Let us just quote here the Chern-Simons kinetic term

$$\frac{1}{2}\varepsilon^{\mu\nu\lambda}\left[f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right], \tag{7}$$

which will be essential in identifying the gauge group structure.

It is $\mathcal{N} = 6$ orbifold $\mathbb{C}^4/\mathbb{Z}_k$ which was considered in Ref. [19]. The spacetime symmetry is combined with translation along σ_3, and in the end one keeps only the modes with momentum number $\alpha_I = (1, 1, -1, -1)$ along σ_3 for Z^I. It can be facilitated also if we formally demand invariance for all k, i.e.

$$Z^I(\sigma_1, \sigma_2, \sigma_3) = e^{2\pi i\alpha_I/k} Z^I(\sigma_1, \sigma_2, \sigma_3 - 2\pi/k) \quad \text{for all } k > 0. \tag{8}$$

Note that the internal space T^3 is now effectively reduced to T^2. It is convenient to denote for instance as $Z^1_{\vec{m}} \equiv Z^1_{(\vec{m},1)}$, $A^\mu_{\vec{m}\vec{n}} \equiv A^\mu_{(\vec{m},1),(\vec{n},-1)}$, where $\vec{m}, \vec{n} \in \mathbb{Z}^2$.

Now the action should be expressed as a (multiple) summation over these $\mathbb{Z}^2$ indices, which can be regularized in terms of large matrices. For $N \times N$ matrices, one first introduces clock and shift matrices ($\xi = e^{2\pi i/N}$)

$$UV = \xi VU, \quad U^N = V^N = 1. \tag{9}$$

Basis vectors are defined and their multiplication is easily computed as follows

$$J^{\vec{n}} = \xi^{-\frac{n_1 n_2}{2}} U^{n_1} V^{n_2}, \qquad J^{\vec{m}} J^{\vec{n}} = \xi^{\vec{m}\times\vec{n}/2} J^{\vec{m}+\vec{n}}. \tag{10}$$

In order to regularize the infinite dimensional 3-algebra action, we construct the matrix fields as

$$Z^I = \frac{1}{\sqrt{N}} \sum_{\vec{m}} Z^I_{\vec{m}} J^{\vec{m}}, \qquad A^\pm_\mu = \frac{4\pi i}{kN} \sum_{\vec{p},\vec{q}} \xi^{\pm\vec{p}\times\vec{q}/2} A_{\mu\vec{p}\vec{q}} J^{\vec{p}+\vec{q}}. \tag{11}$$

Then one can verify [19] that the large-N limit of ABJM model with $U(N) \times U(N)$ gauge group and four bifundamental chiral multiplets is equivalent to the truncated 3-algebra theory through Eq. (11). The original coupling constant κ in that case is given in terms of gauge theory parameters as

$$\kappa = -\frac{4\pi^2}{kN^2}. \tag{12}$$

III. $\mathcal{N} = 4$ ORBIFOLD $\mathbb{C}^2/\mathbb{Z}_k \times \mathbb{C}^2$

Let us now move to our main interest, $\mathbb{C}^2/\mathbb{Z}_k \times \mathbb{C}^2$. The discrete symmetry action is defined on $\mathbb{C}^4$ as

$$(Z^1, Z^2, Z^3, Z^4) \longrightarrow (\omega Z^1, \omega^{-1} Z^2, Z^3, Z^4), \tag{13}$$

where $\omega = e^{2\pi i/k}$. In this background the M2-brane theory should preserve $\mathcal{N} = 4$, i.e. half of the maximal supersymmetry. As before we start with the BL theory with Nambu bracket on T^3, and restrict the momentum modes. The above transformation will be combined with $\mathbb{Z}_k$ translation in the internal space. With $\alpha_I = (1, -1, 0, 0)$, we demand

$$Z^I(\sigma_1, \sigma_2, \sigma_3) = e^{2\pi\alpha_I/k} Z^I(\sigma_1, \sigma_2, \sigma_3 - 2\pi/k), \quad \text{for all } k > 0. \tag{14}$$

We thus fix the momentum modes of Z^I along σ_3 to be α_I, and write as

$$Z^I(x^\mu; \sigma_i) = \sum_{\vec{m}} \mathcal{Z}^I_{\vec{m}}(x^\mu) e^{i\vec{m}\cdot\vec{\sigma}} e^{i\alpha_I \sigma_3}. \tag{15}$$

where $\vec{\sigma} = (\sigma_1, \sigma_2)$. One can again construct four matrix fields using $\mathcal{Z}^I_{\vec{m}}$ as the expansion coefficients for the basis vector $J^{\vec{m}}/\sqrt{N}$. Obviously Z^3, Z^4 should take a different representation than Z^1, Z^2, so we introduce a new symbol and write $\Phi_1 = \mathcal{Z}^3_{\vec{m}} J^{\vec{m}}/\sqrt{N}$, $\Phi_2 = \mathcal{Z}^4_{\vec{m}} J^{\vec{m}}/\sqrt{N}$.

For the consistency we should keep the same type of modes for the gauge field $A^\mu_{\vec{M}\vec{N}}$. Since the covariant derivative should not violate the orbifold projection, we have two surviving sets: one is $(M_3, N_3) = (1, -1)$ or $(-1, 1)$, and the other possibility is $(0, 0)$. We introduce double-index gauge fields as

$$A^{(1)}_{\vec{m}\vec{n}} = A_{(\vec{m},1)(\vec{n},-1)}, \quad A^{(0)}_{\vec{m}\vec{n}} = A_{(\vec{m},0)(\vec{n},0)}. \tag{16}$$

One can show $A^{(1)*}_{\vec{m}\vec{n}} = A^{(1)}_{-\vec{m},-\vec{n}}$ and $A^{(0)*}_{\vec{m}\vec{n}} = +A^{(0)}_{-\vec{m},-\vec{n}}$, from antisymmetry and reality condition of $A_{\vec{M}\vec{N}}$.

In order to identify the gauge symmetry of the truncated and regularized 3-algebra action, let us first consider the Chern-Simons type kinetic term, given above in Eq. (7). One can easily verify

$$\begin{aligned}\frac{1}{2}\varepsilon^{\mu\nu\lambda} f^{abcd} A_{\mu ab}\partial_\nu A_{\lambda cd} \;=\; -4i\kappa \sum_{\vec{m}+\vec{n}+\vec{p}+\vec{q}=0} \Big[&(\vec{p}\times\vec{q}) A^{(1)}_{\vec{m}\vec{n}} \mathrm{d}A^{(0)}_{\vec{p}\vec{q}} \\ &+ (\vec{m}\times\vec{n} + \vec{p}\times\vec{q}) A^{(1)}_{\vec{m}\vec{n}} \mathrm{d}A^{(1)}_{\vec{p}\vec{q}} \Big].\end{aligned} \tag{17}$$

We again want to construct matrix fields using the 3-algebra expansion coefficients $A^{(I)}_{\vec{p}\vec{q}}$. As a start, let us define matrix gauge fields in the same way as we did for $\mathbb{C}^4/\mathbb{Z}_k$.

$$A^{\pm(I)}_\mu = \frac{4\pi i}{kN} \sum \xi^{\pm\vec{p}\times\vec{q}/2} A^{(I)}_{\mu\vec{p}\vec{q}} J^{\vec{p}+\vec{q}}, \quad I = 0, 1. \tag{18}$$

In fact one can show $A^{+(0)} = -A^{-(0)}$, using $A^{(0)}_{\vec{m}\vec{n}} = -A^{(0)}_{\vec{n}\vec{m}}$. Now we consider some products of matrix fields and get the following results:

$$\mathrm{Tr}(A^{(1)}_+ \mathrm{d}A^{(1)}_+ - A^{(1)}_- \mathrm{d}A^{(1)}_-) \;=\; -\frac{32\pi^3 i}{k^2N^2} \sum_{\vec{m}+\vec{n}+\vec{p}+\vec{q}=0} (\vec{m}\times\vec{n} + \vec{p}\times\vec{q}) A^{(1)}_{\vec{m}\vec{n}}\, \mathrm{d}A^{(1)}_{\vec{p}\vec{q}} \tag{19}$$

$$\mathrm{Tr}\,(A^{(1)}_+ + A^{(1)}_-)\,\mathrm{d}A^{(0)}_+ \;=\; -\frac{32\pi^3 i}{k^2N^2} \sum_{\vec{m}+\vec{n}+\vec{p}+\vec{q}=0} (\vec{p}\times\vec{q})\, A^{(1)}_{\vec{m}\vec{n}}\, \mathrm{d}A^{(0)}_{\vec{p}\vec{q}} \tag{20}$$

Then it is easy to see that, if we define new matrix fields as

$$\tilde{A}^\pm = A^{\pm(1)} + \tfrac{1}{2}A^{\pm(0)}, \tag{21}$$

Eq. (17) is reproduced by

$$\tfrac{k}{4\pi}\mathrm{Tr}(\tilde{A}_+ \mathrm{d}\tilde{A}_+ - \tilde{A}_- \mathrm{d}\tilde{A}_-) \tag{22}$$

in the large N limit, if we again demand Eq. (12).

Compared to the $\mathcal{N}=6$ case, the truncation seems to give us one more gauge field $A^{(0)}$, but rather surprisingly the study of gauge kinetic term suggests we still have $U(N)\times U(N)$ gauge groups with Chern-Simons level k and $-k$ respectively. Of course we need to check whether this simplification persists with the gauge field cubic interaction term. The second term in Eq. (7) is reduced by the truncation into

$$4\kappa^2 \sum_{\vec{m}+\vec{n}+\vec{p}+\vec{q}+\vec{r}+\vec{s}=0} \left[(\vec{m}+\vec{n})\times(\vec{p}+\vec{q})\right](\vec{r}\times\vec{s}) A^{(1)}_{\vec{m}\vec{n}} A^{(1)}_{\vec{p}\vec{q}} \left(A^{(1)}_{\vec{r}\vec{s}} + A^{(0)}_{\vec{r}\vec{s}}/2\right). \tag{23}$$

Let us then consider the cubic term which would be consistent with Eq. (22) on the matrix side:

$$\begin{aligned}\mathrm{Tr}(\tilde{A}_+^3 - \tilde{A}_-^3) \;=\; & \mathrm{Tr}\left(A_+^{(1)3} - A_-^{(1)3}\right) + \tfrac{3}{2}\mathrm{Tr}\, A_+^{(0)}\left(A_+^{(1)2} + A_-^{(1)2}\right) \\ & + \tfrac{3}{4}\mathrm{Tr} A_+^{(0)2}\left(A_+^{(1)} - A_-^{(1)}\right) + \tfrac{1}{4}\mathrm{Tr} A_+^{(0)3}.\end{aligned} \tag{24}$$

One can easily see that, in the large-N limit the first two terms are $\mathcal{O}(1/N)$, while the last two terms are $\mathcal{O}(1/N^3)$ and thus negligible in the above expression. Now it is rather tedious but straightforward to show that $\frac{2i}{3}\frac{k}{4\pi}\mathrm{Tr}(\tilde{A}_+^3 - \tilde{A}_-^3)$ in the large-N limit is equivalent to Eq. (23), if we accept Eq. (12). One can thus establish that for our $\mathcal{N}=4$ orbifold the matrix regularization reduces the Chern-Simons kinetic terms into

$$\tfrac{k}{4\pi}\mathrm{Tr}(\tilde{A}_+ \mathrm{d}\tilde{A}_+ + \tfrac{2i}{3}\tilde{A}_+^3 - \tilde{A}_- \mathrm{d}\tilde{A}_- - \tfrac{2i}{3}\tilde{A}_-^3). \tag{25}$$

We now turn to check if the matter fields have consistent couplings with $\tilde{A}_\pm$. For Z^1(Z^2 is simply the complex conjugate), we have

$$(D_\mu \mathcal{Z}^1)_{\vec{n}} = \partial_\mu \mathcal{Z}^1_{\vec{n}} + i\kappa \sum_{\vec{p}+\vec{q}+\vec{m}=\vec{n}} \left[2\left(\vec{p}\times\vec{q} + (\vec{p}+\vec{q})\times\vec{m}\right) A^{(1)}_{\mu\vec{p}\vec{q}} + (\vec{p}\times\vec{q}) A^{(0)}_{\mu\vec{p}\vec{q}}\right] \mathcal{Z}^1_{\vec{m}}. \tag{26}$$

Let us define a scalar field on the matrix side again as $\mathcal{Z}^1 = \frac{1}{\sqrt{N}}\sum_{\vec{m}} \mathcal{Z}^1_{\vec{m}} J^{\vec{m}}$, with a slight abuse of notation. It is then straightforward to verify that, the matrix field representation

$$D_\mu \mathcal{Z}^1 = \partial_\mu \mathcal{Z}^1 + i(\tilde{A}_{+\mu}\mathcal{Z}^1 - \mathcal{Z}^1 \tilde{A}_{-\mu}) \tag{27}$$

in the large-N limit approaches Eq. (26), with the same identification Eq. (12). This obviously implies $\mathcal{Z}^1$ is in $(\mathbf{N},\bar{\mathbf{N}})$ representation of $U(N)\times U(N)$, while $\mathcal{Z}^2$ is $(\bar{\mathbf{N}},\mathbf{N})$.

The remaining scalar fields Φ^1, Φ^2 on the other hand couple to $A^{(1)}$ only, and the 3-algebra side computation gives

$$(D_\mu \Phi)_{\vec{n}} = \partial_\mu \Phi_{\vec{n}} + 2i\kappa \sum_{\vec{p}+\vec{q}+\vec{m}=\vec{n}} [(\vec{p}+\vec{q}) \times \vec{m}]\, A^{(1)}_{\mu\vec{p}\vec{q}} \Phi_{\vec{m}}. \tag{28}$$

Our objective here is to identify the matrix Chern-Simons-matter theory which approaches the truncated 3-algebra theory in the large-N limit. The correction terms, or the error, have been kept small by $\mathcal{O}(1/N^2)$, compared to the leading order terms. It is natural to require the same with Φ fields, but then the covariant derivative has to be

$$D_\mu \Phi = \partial_\mu \Phi + i \left[\frac{\tilde{A}_{+\mu} + \tilde{A}_{-\mu}}{2} \Phi - \Phi \frac{\tilde{A}_{+\mu} + \tilde{A}_{-\mu}}{2} \right]. \tag{29}$$

It is only with the above expression that we can cancel the contributions from $A^{(0)}$ in $\tilde{A}_\pm$, and correctly reproduce Eq. (28) up to $\mathcal{O}(1/N)$. However it is obvious that the gauge coupling of Φ in Eq. (29) does not exhibit a well-defined transformation rule. In other words, Eq. (28) is not compatible with $U(N) \times U(N)$ gauge symmetry represented by $\tilde{A}_\pm$.

If we insist on consistent gauge symmetry, we may instead choose

$$D_\mu \Phi = \partial_\mu \Phi + i \left(\tilde{A}_{+\mu} \Phi - \Phi \tilde{A}_{+\mu} \right). \tag{30}$$

Then Φ's are in the adjoin representation of $\tilde{A}_+$, but now the correction terms are order $\mathcal{O}(1/N)$! Note that one can substitute $\tilde{A}_+$ with $\tilde{A}_-$ in Eq.(30), and again the terms involving $A^{(0)}_{\vec{m}\vec{n}}$ are smaller by $\mathcal{O}(1/N)$, compared to $A^{(1)}$ terms. One should note the difference with $\mathcal{Z}$ fields, where the coupling is $(\tilde{A}_+ \mathcal{Z} - \mathcal{Z} \tilde{A}_-)$ and the correction terms are $\mathcal{O}(1/N^2)$.

Finally let us consider the superpotential. Using the same matrix parametrization, it is

$$W = \frac{4\pi}{k} \mathrm{Tr}(\mathcal{Z}^1 \mathcal{Z}^2 [\Phi^1, \Phi^2]) \tag{31}$$

which is consistent with the gauge symmetry and approaches Eq. (6) in the large-N limit. Again, the correction terms are relatively $\mathcal{O}(1/N)$.

With the above prescription our orbifolded action becomes equivalent to the quiver gauge theory for membranes on $\mathbb{C}^2/\mathbb{Z}_k \times \mathbb{C}^2$ proposed in Ref.[11].

IV. DISCUSSION

In this article we continued with our proposal [19] for 3-algebra theory and applied it to a $\mathcal{N} = 4$ orbifold. The Nambu-bracket BLG theory has an infinite number of fields,

and its physical relevance is unclear at the quantum level. But it certainly enjoys maximal supersymmetry, so one might fathom it somehow encompasses the multi-membrane theory in flat background. Indeed, it can be shown that when one takes a version of orbifold procedure $\mathbb{C}^4/\mathbb{Z}_k$ the 3-algebra theory becomes equivalent to large-N limit of the ABJM model.

For the $\mathbb{C}^2/\mathbb{Z}_k \times \mathbb{C}^2$ orbifold we have witnessed a problem with matrix regularization. If we express the 3-product using the Poisson bracket in the most natural way, the matrix regularization does not lead to a consistent Yang-Mills invariant theory. One needs to sacrifice the consistency of large-N approximation and allow for $\mathcal{O}(1/N)$ discrepancy which was not present with $\mathbb{C}^4/\mathbb{Z}_k$.

Our approach might explain the puzzle of supersymmetry enumeration with dual ABJM model. Although it is presented as a dual to $\mathcal{N}=4$ orbifold, the field theory model has only $\mathcal{N}=3$. In our analysis, the reduced 3-algebra description, written for instance using Eqs. (17),(23),(26), and (28) must be $\mathcal{N}=4$. But the 3-algebra side action corresponds to the large-N limit of dual ABJM model, at best. It should be the mismatch of orders of magnitude for subleading terms which break the supersymmetry to $\mathcal{N}=3$.

Finally, one might ask why we have an ambiguity in identifying the gauge symmetry. After all, the BLG theory is invariant under 3-algebra transformation. We can think of two technical points. First, the 3-algebra symmetry is presented only with infinitesimal transformation in Ref. [2, 3, 4]: unlike Yang-Mills symmetry, there is no analogue of multiplication by generic unitary matrix or finite transformation *per se*. This problem is especially acute when we see Eq. (29), which would suffer from non-commutativity of unitary transformations for $\tilde{A}_+$ and $\tilde{A}_-$. Second, the supersymmetry and 3-algebra transformation rules of BLG theory are given for $\tilde{A}_\mu{}^b{}_a = f^{cdb}{}_a A_{\mu cd}$, while the action itself is given in terms of $\tilde{A}_\mu{}^b{}_a$. For $SO(4)$ one can immediately invert and express the transformation rules for $\tilde{A}_\mu{}^b{}_a$, but with Eq. (4) the inversion is far from clear.

Acknowledgments

The research of N. Kim is supported by the Science Research Center Program of the Korea Science and Engineering Foundation through the Center for Quantum Spacetime (CQUeST) of Sogang University with grant number R11-2005-021, and also partly by the

Korea Research Foundation Grant No. KRF-2007-331-C00072.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998), hep-th/9711200.

[2] J. Bagger and N. Lambert, Phys. Rev. **D75**, 045020 (2007), hep-th/0611108.

[3] J. Bagger and N. Lambert, Phys. Rev. **D77**, 065008 (2008), 0711.0955.

[4] J. Bagger and N. Lambert, JHEP **02**, 105 (2008), 0712.3738.

[5] A. Gustavsson, Nucl. Phys. **B811**, 66 (2009), 0709.1260.

[6] R. C. Myers, JHEP **12**, 022 (1999), hep-th/9910053.

[7] G. Papadopoulos, JHEP **05**, 054 (2008), 0804.2662.

[8] J. P. Gauntlett and J. B. Gutowski, JHEP **06**, 053 (2008), 0804.3078.

[9] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, JHEP **10**, 091 (2008), 0806.1218.

[10] Y. Imamura and K. Kimura, Prog. Theor. Phys. **120**, 509 (2008), 0806.3727.

[11] A. Hanany, D. Vegh, and A. Zaffaroni, JHEP **03**, 012 (2009), 0809.1440.

[12] A. Hanany and A. Zaffaroni, JHEP **10**, 111 (2008), 0808.1244.

[13] Y. Imamura and K. Kimura, JHEP **10**, 114 (2008), 0808.4155.

[14] S. Lee, Phys. Rev. **D75**, 101901 (2007), hep-th/0610204.

[15] J.-H. Park and C. Sochichiu, Eur. Phys. J. **C64**, 161 (2009), 0806.0335.

[16] I. Jeon, J. Kim, B.-H. Lee, J.-H. Park, and N. Kim (2008), 0809.0856.

[17] I. Jeon, J. Kim, N. Kim, S.-W. Kim, and J.-H. Park, JHEP **07**, 056 (2008), 0805.3236.

[18] P.-M. Ho and Y. Matsuo, JHEP **06**, 105 (2008), 0804.3629.

[19] N. Kim (2008), 0807.1349.

[20] D. Rodriguez-Gomez (2009), 0911.0008.

[21] J. Choi, S. Lee, and J. Song, JHEP **03**, 099 (2009), 0811.2855.